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Innovation and Competition in a Memory Process

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Innovation & Competition in a Memory Process

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Abstract

Does innovation boost or fall with more competition when innovation follows a memory process? This paper provides a theoretical model which analyzes the innovation and competition relationship assuming that innovation follows a short-memory process. I find innovation increases with more product market competition, even under unpriced spillovers. Assuming the probability to innovate increases with past innovations; a follower firm has large incentives to innovate, even in a highly competitive environment, since the memory obtained after innovating increases its probability to innovate again and become a leader. Therefore, industries will be most of the time neck-and-neck where firms innovate to escape from competition. Using the same dataset of Aghion et al. (2005) I also find there is a positive empirical relationship between innovation and competition. In the case of memoryless industries, I show there is no significant relationship between innovation and competition.

Keywords: innovation, R&D, competition, memory process
JEL Classification: D43, L11, O31

1 Introduction

This paper proposes a model concerning the relationship between innovation and competition, assuming a firm's current probability to innovate increases when this firm succeeded to innovate in the past. I theoretically find that the higher the competition's level, the

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more innovative an industry is. Using the same dataset of Aghion, Bloom, Blundell, Griffith and Howitt (2005), hereafter ABBGH, I find a positive empirical relationship between innovation and competition.

I introduce a theoretical model assuming the contemporaneous probability of innovation success depends on the previous period's innovation success. In this model, as in ABBGH, there is a negative relationship between innovation and competition in unleveled industries, while this relationship is positive in neck-and-neck industries. However, since the incentive to innovate in unleveled industries is still very high at the maximum level of competition, the industries will be most of the time neck-and-neck, where the relationship between innovation and competition is upward sloping; differing from the inverted-U relationship found in ABBGH. I also find that more memory increases the level of research intensity.

This theoretical model is tested using the ABBGH dataset. Across the entire sample, ABBGH find an inverted-U relationship between competition and innovation. I show, however, that there is a structural break in the data, in 1983. Splitting the sample between 1973-1982 and 1983-1994, the empirical relationship between innovation and competition is positive for the industries which show a memory process during the period 1973-1982. However, there is no significant relationship (either positive or negative) between innovation and competition during the period 1983-1994. The inverted-U pattern does not hold for the memoryless either, since in fact there is no significant relationship between innovation and competition for these industries.

Hence, the data before 1983 provide some support for a model in which the innovation process has memory; but clearly, the period after 1983 is quite different. I do not attempt to give a full explanation of the structural break in this paper. It is noted, however, that Jaffe and Lerner (2004) argue that the establishment of the United States Court of Appeals for the Federal Circuit (CAFC) in October 1982 decreased the difficulty of granting a patent, diminishing the quality of those which are granted. The ABBGH dataset is made up of firms listed on the London Stock Exchange that received patent grants from the United States Patent and Trademark Office. One hypothesis, then, is that this change in the US institutional environment had an effect on the relationship between innovation and competition.

Despite a long and large literature, there is still debate about whether competition boosts or discourages innovation. This paper's finding suggests a positive innovation-competition relationship when firms learn from their own innovations, adding in the discussion that the memory assumption matters.

Some theoretical models, such as Aghion and Howitt (1992), predict that more competition tends to decrease innovation activity. In these models, the innovation incentive is represented by the difference between the expected value of post-innovation profit and pre-innovation profit. Since a monopolist would replace her/himself whenever she/he creates a better intermediate good, incentive to innovate would be biased toward the entrant. Should the profit of being a monopoly be small, as in the case when there is high level of competition, entrants have little incentive to innovate. In contrast, Boldrin and Levine (2008) show that competition leads to more innovation. In their model it is assumed that any idea is costly to transmit, thus there are no unpriced spillovers.

Until recently, a number of empirical studies, such as Geroski (1990), find an empirical positive relationship between innovation and competition[1].

Nevertheless, ABBGH find an empirical inverted-U relationship between competition and innovation. This pattern is explained by their theoretical model, which states when there is very low competition, profits before and after innovating are not so different, hence there is not too much incentive to innovate when firms have similar costs (neck-and-neck sector). In an unleveled sector, if the leader innovates the follower keeps the technology distance because of spillovers, thus the leader does not have incentive to innovate. Due to that, low competition industries will be most of the time neck-and-neck where any increase in the competition level leads firms to innovate in order to escape from it (*escape-competition effect*). When there is too much competition, the follower's profit after innovating is not so different to pre-innovation profit, consequently there is not much incentive to innovate in the unleveled case. For neck-and-neck firms, any innovation would give more profit than if they remain leveled. Therefore, too high competition leads industries to be unleveled most of the time, while any decrease in the level of competition boosts the follower's profit if it innovates (*Schumpeterian effect*).

One important issue about ABBGH model is it does not take into account that industries can exhibit memory in the process of innovation. Hausman et al. (1984) and Hall et al. (1986) find that contemporaneous research intensity is correlated with past research intensity, giving support to the memory assumption.

The rest of the article is organized as follows. Section 2 proposes a theoretical model which assumes that firms follow a short-memory process. Section 3 develops the methodology to define whether the innovation process has memory or is memoryless, and to show the empirical relationship between innovation and competition for both memory and memoryless industries. Section 4 concludes.

[1]Geroski (1990) uses the extent of market penetration by entrants, the market share of imports, the relative number of small firms (measured by number of employees), the within period percentage change in concentration, the market share of exiting firms and the five firm market concentration ratio, as indicators of rivalry.

2 Theoretical Memory Model

Following ABBGH I assume there are two firms, where $n_m^{k,l}$ denotes the research intensity of a firm which is m technological steps away from its competitor at time t and was k steps away from its rival at time $t-\Delta t$; and l takes the value S if the firm succeeded to innovate at time $t-\Delta t$ or F if it failed. As in ABBGH, a firm cannot be more than one step ahead of its opponent, since the latter can copy the leader's previous technology whenever it innovates. Therefore, the leader's research intensity is zero as it cannot obtain additional value from innovating. Consequently, a firm can be in three possible levels, one step ahead, neck-and-neck and one step behind, these states are denoted by the sub(super)scripts 1, 0 and -1 respectively.

As in ABBGH and Aghion et al. (2001), h represents the copy rate or R&D spillovers. The leader obtains profit π_1 and neck-and-neck π_0, while the follower does not gain any profit. Neck-and-neck's profit π_0 can also be expressed as $\pi_1(1-\delta), \forall\delta \in [0.5, 1]$, where δ is the product market competition parameter. The higher the value of δ, the more competitive the industry is. Assuming Bertrand competition, the firms equally share the monopoly profits, π_1, with maximum level of collusion $\delta = 0.5$. By the contrary, firms receive zero profits when competition reaches its maximum level at $\delta = 1$. The research cost of a firm is given by $\frac{(n_m^{k,l})^2}{2}$. The research intensity of the rival firm when they are neck-and-neck is denoted as $\overline{n}_0^{k,l}$ and a symmetric Nash equilibrium turns to $n_0^{k,l} = \overline{n}_0^{k,l}$.

2.1 Bellman and research equations

The model presented in this section is constructed assuming that changes in the relative position (leader, neck-and-neck and follower) can only occur after innovating. Hausman et al. (1984) and Hall et al. (1986) empirically find that contemporaneous R&D investment depends on previous investment. Therefore, I assume whenever a firm improves its relative position at time $t-\Delta t$, a value λ, which increases the probability of innovating at time t, will be generated under the process, where $\lambda > 0$. When a firm innovates, but does not improve its relative position, it obtains a value, ϕ, which also increases the probability of innovating at time t; where $\phi > 0$. After time t, both λ and ϕ depreciate completely, i.e. memory lasts for only one period; after that, any advantage completely disappears. The memory values of rival firms are denoted as $\overline{\lambda}$ and $\overline{\phi}$.

The value function of each firm is given by the current profit flow, the discounted expected value of the firm after investing in R&D and the cost of investing in R&D. The discount factor $e^{-r\Delta t}$ can be expressed as $(1-\Delta t)$ for Δt to be small. From this, we can also have that $(\Delta t)^2 \approx 0$.

For the case of unleveled industries, the leader profits flow is $\pi_1 \Delta t$, while the follower's profit is zero. Since the leader does not invest in R&D, the expected value of an unleveled firm after the follower invest in R&D is given by the follower's probability to innovate $(n_{-1}^{k}+h)\Delta t$. For the same reason only the follower incurs in R&D cost. After the follower invest in R&D there are two possible outcomes: (i) both the leader and the follower continue in the same position with value functions V_1^1 and V_{-1}^{-1} respectively, or (ii) the leader is caught up by the follower with value functions V_0^1 and V_0^{-1} respectively.

Therefore, the value function for the leader who was a leader in the previous period can be written as

$$V_1^1 = \pi_1 \Delta t + (1 - r\Delta t)\left[(\overline{n}_{-1}^{-1} + h)\Delta t V_0^1 + [1 - (\overline{n}_{-1}^{-1} + h)\Delta t]V_1^1\right];$$

for the leader who was neck-and-neck in the previous period

$$V_1^0 = \pi_1 \Delta t + (1 - r\Delta t)\left[(\overline{n}_{-1}^{0} + h)\Delta t V_0^1 + [1 - (\overline{n}_{-1}^{0} + h)\Delta t]V_1^1\right];$$

for the follower who was follower in the previous period

$$V_{-1}^{-1} = \max_{n_{-1}^{-1}}\left\{(1 - r\Delta t)\left[(n_{-1}^{-1} + h)\Delta t V_0^{-1} + [1 - (n_{-1}^{-1} + h)\Delta t]V_{-1}^{-1}\right] - \frac{(n_{-1}^{-1})^2}{2}\Delta t\right\};$$

and for the follower who was neck-and-neck in the previous period

$$V_{-1}^{0} = \max_{n_{-1}^{0}}\left\{(1 - r\Delta t)\left[(n_{-1}^{0} + h)\Delta t V_0^{-1} + [1 - (n_{-1}^{0} + h)\Delta t]V_{-1}^{0}\right] - \frac{(n_{-1}^{0})^2}{2}\Delta t\right\}.$$

For neck-and-neck industries, the firm's profit flow is $\pi_0\Delta t$. The expected value of a neck-and-neck firm, which was also neck-and-neck in the previous period and failed to innovate, is a function of its own research intensity $n_0^{0,F}$ and the rival's research intensity $\overline{n}_0^{0,F}$. In the case that the neck-and-neck firm succeeded to innovate in the previous period, the expected value is a function of the research intensities $n_0^{0,S}$ and $\overline{n}_0^{0,S}$, and the memory parameters ϕ and $\overline{\phi}$ obtained in the innovation processes. For neck-and-neck firms which were unleveled in the previous period, the expected value of the firms is a function of the research intensities and the memory parameter λ obtained by the firm which was a follower the previous period. For any of the neck-and-neck firm there are four possible outcomes after they invest in R&D: (i) Succeeding to innovate while the rival fails with a value function V_1^0, (ii) Failing to innovate while the rival succeeds with a value function V_{-1}^0, (iii) both succeeding to innovate with a value function $V_0^{0,S}$ and (iv) both failing to innovate with a value function $V_0^{0,F}$.

Therefore, the value function for the neck-and-neck firm that was also neck-and-neck in the previous period and failed to innovate, as well as its rival, is

$$\begin{aligned} V_0^{0,F} &= \max_{n_0^{0,F}} \Big\{ \pi_0 \Delta t + (1 - r\Delta t) \Big[n_0^{0,F} \Delta t V_1^0 + \overline{n}_0^{0,F} \Delta t V_{-1}^0 + (n_0^{0,F} + \overline{n}_0^{0,F}) \Delta t V_0^{0,S} \\ &\quad + [1 - (2n_0^{0,F} + 2\overline{n}_0^{0,F}) \Delta t] V_0^{0,F} \Big] - \frac{(n_0^{0,F})^2}{2} \Delta t \Big\}; \end{aligned}$$

for the neck-and-neck firm, which was also a neck-and-neck in the previous period but succeeded to innovate, i.e. both firms innovated in the previous period,

$$\begin{aligned} V_0^{0,S} &= \max_{n_0^{0,S}} \Big\{ \pi_0 \Delta t + (1 - r\Delta t) \Big[(n_0^{0,S} + \phi) \Delta t V_1^0 + (\overline{n}_0^{0,S} + \overline{\phi}) \Delta t V_{-1}^0 \\ &\quad + (n_0^{0,S} + \phi + \overline{n}_0^{0,S} + \overline{\phi}) \Delta t V_0^{0,S} + [1 - (2n_0^{0,S} + 2\phi + 2\overline{n}_0^{0,S} + 2\overline{\phi}) \Delta t] V_0^{0,F} \Big] - \frac{(n_0^{0,S})^2}{2} \Delta t \Big\}; \end{aligned}$$

for the one which was a follower in the previous period

$$\begin{aligned} V_0^{-1} &= \max_{n_0^{-1}} \Big\{ \pi_0 \Delta t + (1 - r\Delta t) \Big[(n_0^{-1} + \lambda) \Delta t V_1^0 + \overline{n}_0^1 \Delta t V_{-1}^0 + (n_0^{-1} + \lambda + \overline{n}_0^1) \Delta t V_0^{0,S} \\ &\quad + [1 - (2n_0^{-1} + 2\lambda + 2\overline{n}_0^1) \Delta t] V_0^{0,F} \Big] - \frac{(n_0^{-1})^2}{2} \Delta t \Big\}; \end{aligned}$$

and for the one which was a leader

$$\begin{aligned} V_0^1 &= \max_{n_0^1} \Big\{ \pi_0 \Delta t + (1 - r\Delta t) \Big[n_0^1 \Delta t V_1^0 + (\overline{n}_0^{-1} + \overline{\lambda}) \Delta t V_{-1}^0 + (n_0^1 + \overline{n}_0^{-1} + \overline{\lambda}) \Delta t V_0^{0,S} \\ &\quad + [1 - (2n_0^1 + 2\overline{n}_0^{-1} + 2\overline{\lambda}) \Delta t] V_0^{0,F} \Big] - \frac{(n_0^1)^2}{2} \Delta t \Big\}. \end{aligned}$$

After simplifying the value functions, the annuity values can be expressed as

$$rV_1^1 = \pi_1 + (\overline{n}_{-1}^{-1} + h)(V_0^1 - V_1^1);$$

$$rV_1^0 = r\Delta t \left[\pi_1 + (\overline{n}_{-1}^{-1} + h)(V_0^1 - V_1^1) \right] + (1 - r\Delta t) rV_1^1;$$

$$rV_{-1}^{-1} = \max_{n_{-1}^{-1}} \left\{ (n_{-1}^{-1} + h)(V_0^{-1} - V_{-1}^{-1}) - \frac{(n_{-1}^{-1})^2}{2} \right\};$$

$$rV_{-1}^0 = \max_{n_{-1}^0} \left\{ r\Delta t \left[(n_{-1}^0 + h)(V_0^{-1} - V_{-1}^{-1}) - \frac{(n_{-1}^{-1})^2}{2} \right] + (1 - r\Delta t) rV_{-1}^{-1} \right\};$$

$$rV_0^{0,F} = \max_{n_0^{0,F}} \left\{ \pi_0 + n_0^{0,F} (V_1^0 + V_0^{0,S} - 2V_0^{0,F}) + \overline{n}_0^{0,F} (V_{-1}^0 + V_0^{0,S} - 2V_0^{0,F}) - \frac{(n_0^{0,F})^2}{2} \right\};$$

$$
\begin{aligned}
rV_0^{0,S} &= \max_{n_0^{0,S}}\Big\{r\Delta t\Big[\pi_0 + (n_0^{0,S}+\phi)(V_1^0 + V_0^{0,S} - 2V_0^{0,F}) + (\overline{n}_0^{0,S}+\overline{\phi})(V_{-1}^0 + V_0^{0,S} \\
&\quad -2V_0^{0,F}) - \frac{(n_0^{0,S})^2}{2}\Big] + (1-r\Delta t)rV_0^{0,F}\Big\};
\end{aligned}
$$

$$
\begin{aligned}
rV_0^{-1} &= \max_{n_0^{-1}}\Big\{r\Delta t\Big[\pi_0 + (n_0^{-1}+\lambda)(V_1^0 + V_0^{0,S} - 2V_0^{0,F}) + \overline{n}_0^1(V_{-1}^0 + V_0^{0,S} - 2V_0^{0,F}) \\
&\quad -\frac{(n_0^{-1})^2}{2}\Big] + (1-r\Delta t)rV_0^{0,F}\Big\};
\end{aligned}
$$

$$
\begin{aligned}
rV_0^{1} &= \max_{n_0^{1}}\Big\{r\Delta t\Big[\pi_0 + n_0^{1}(V_1^0 + V_0^{0,S} - 2V_0^{0,F}) + (\overline{n}_0^{-1}+\overline{\lambda})(V_{-1}^0 + V_0^{0,S} - 2V_0^{0,F}) \\
&\quad -\frac{(n_0^{1})^2}{2}\Big] + (1-r\Delta t)rV_0^{0,F}\Big\}.
\end{aligned}
$$

First-order conditions can be formulated as

$$n_{-1}^{-1} = V_0^{-1} - V_{-1}^{-1}; \tag{1}$$

$$n_{-1}^{0} = V_0^{-1} - V_{-1}^{-1}; \tag{2}$$

$$n_0^{0,F} = V_1^0 + V_0^{0,S} - 2V_0^{0,F}; \tag{3}$$

$$n_0^{0,S} = V_1^0 + V_0^{0,S} - 2V_0^{0,F}; \tag{4}$$

$$n_0^{-1} = V_1^0 + V_0^{0,S} - 2V_0^{0,F}; \tag{5}$$

$$n_0^{1} = V_1^0 + V_0^{0,S} - 2V_0^{0,F}. \tag{6}$$

From (1) and (2) we can see that the research intensity of a follower which was a follower in the previous period is the same one of the follower which was neck-and-neck. This is due to the fact that the leader does not innovate and it is not possible to be a follower after innovating, thus the value function of the leader is the same whether it was a leader or neck-and-neck the previous period and the value function of the follower is the same whether it was a follower or neck-and-neck the previous period.

From (3), (4), (5) and (6); we can also observe that each type of neck-and-neck firms has the same level of research intensity. This is because the knowledge, λ or ϕ, obtained in the previous period, is independent of the research intensity in the current time, thus it does not affect the marginal decision to research in the actual period. Therefore, notation is simplified as $n_{-1}^{-1} = n_{-1}^{0} \equiv n_{-1}$ and $n_0^{0,F} = n_0^{0,S} = n_0^{-1} = n_0^1 \equiv n_0$.

After rearranging, we can see a system of two equations and two variables (the research intensities), which can be written as follows

$$0 = \frac{1-\phi\Delta t}{1-2\phi\Delta t}(V_1 - V_0^{0,F}) - \frac{\phi\Delta t}{1-2\phi\Delta t}(V_0^{0,F} - V_{-1}) - n_0;$$

$$0 = \pi_1(1-\delta)\Delta t + (1-r\Delta t)V_0^{0,F} - n_0\Delta t(V_1 - V_{-1}) + \frac{3}{2}n_0\Delta t + \lambda n_0\Delta t - \frac{r+h}{r}n_{-1} - \frac{n_{-1}^2}{2r};$$

where

$$\begin{aligned} V_0^1 &= \frac{r+h+n_{-1}}{r(r+h+n_{-1}) + (n_0+\lambda\Delta t)(n_{-1}+h)}\Bigg[\pi_1(1-\delta) - \frac{\pi_1(\lambda\Delta t + n_0)}{r+h+n_{-1}} \\ &\quad + \frac{3}{2}n_0^2 + \frac{\lambda n_{-1}^2\Delta t}{2r} + \lambda n_0\Delta t + n_{-1}\frac{r+h-r\lambda\Delta t}{r} + n_0(\frac{n_{-1}^2}{2r} + n_{-1}\frac{r+h}{r} - n_{-1})\Bigg]; \end{aligned}$$

$$V_0^{0,F} = \frac{1}{r}\left\{\pi_1\left[(1-\delta) - \frac{n_0}{r+h+n_{-1}}\right] + \frac{3}{2}n_0^2 + n_0(\frac{n_{-1}^2}{r} + n_{-1}\frac{r+h}{r} - n_{-1}) - \frac{n_0(n_{-1}+h)}{r+h+n_{-1}}V_0^1\right\};$$

$$V_1 = \frac{\pi_1 + (n_{-1}+h)V_0^1}{r+h+n_{-1}};$$

$$V_{-1} = \frac{n_{-1}}{r}(\frac{n_{-1}}{2} + h).$$

By the complexity of these equations we can notice that it is not possible to have analytical solutions for the research intensities n_{-1} and n_0. Therefore, I proceed to solve the steady state and then to have a numerical solution for both the research equations and the aggregate flow of innovations.

2.2 Steady state

In this model, as we can see in figure 1, there are four different states. The firms can be unleveled with probability μ_1; neck-and-neck, having been neck-and-neck and failed to innovate in the previous period, with probability $\mu_0^{0,F}$; neck-and-neck, having been neck-and-neck and succeeded to innovate in the previous period, with probability $\mu_0^{0,S}$; and neck-and-neck, having been unleveled in the previous period, with probability μ_0^1.

An unleveled industry can become a neck-and-neck after the follower innovates, thus the outflow is equal to $(n_{-1}+h)\Delta t\mu_1$. Since the state of an unleveled industry can be originated by a neck-and-neck state which was neck-and-neck and failed to innovate in the previous period, a neck-and-neck which was neck-and-neck and succeeded to innovate in the previous period or a neck-and-neck that was unleveled in the previous period, the inflows of this case are given by $2n_0\Delta t\mu_0^{0,F} + 2(n_0+\phi)\Delta t\mu_0^{0,S} + (2n_0+\lambda)\Delta t\mu_0^1$.

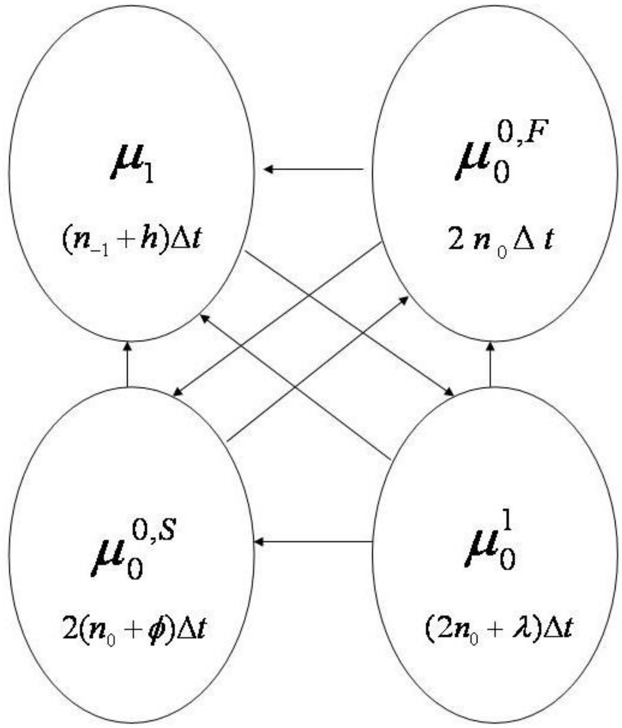


Figure 1: Steady State

A neck-and-neck industry, which was also neck-and-neck and failed to innovate in the previous period, can continue to be neck-and-neck after both firms innovate or become unleveled, thus the outflows are $4n_0\Delta t\mu_0^{0,F}$. In order to become a neck-and-neck that was neck-and-neck and failed to innovate in the past, industries must have been neck-and-neck, hence the inflows are $2(n_0+\phi)\Delta t\mu_0^{0,S}+(2n_0+\lambda)\Delta t\mu_0^1$.

A neck-and-neck industry, which was neck-and-neck and succeeded to innovate in the previous period, can continue to be neck-and-neck after both fail to innovate or become unleveled, hence the outflows are $4(n_0+\phi)\Delta t\mu_0^{0,S}$. To become a neck-and-neck that was neck-and-neck and succeeded to innovate in the previous period, industries must have been neck-and-neck, thus the inflows are $2n_0\Delta t\mu_0^{0,F}+(2n_0+\lambda)\Delta t\mu_0^1$.

Finally, a neck-and-neck industry, which was unleveled before, can become to an unleveled or a neck-and-neck state which was neck-and-neck and either succeeded or failed to innovate in the previous period, thus the outflows of this state are $3(2n_0+\lambda)\Delta t\mu_0^1$. Since to become a neck-and-neck having been unleveled before can only be produced by an unleveled state, the inflow is $(n_{-1}+h)\Delta t\mu_1$.

In the steady state the outflows must be equal to the inflows, thus we have the following equations

$$\mu_1(n_{-1}+h) = 2n_0\mu_0^{0,F} + 2(n_0+\phi)\mu_0^{0,S} + (2n_0+\lambda)\mu_0^1;$$

$$4n_0\mu_0^{0,F} = 2(n_0+\phi)\mu_0^{0,S} + (2n_0+\lambda)\mu_0^1; \tag{7}$$

$$4(n_0+\phi)\mu_0^{0,S} = 2n_0\mu_0^{0,F} + (2n_0+\lambda)\mu_0^1; \tag{8}$$

$$3(2n_0+\lambda)\mu_0^1 = (n_{-1}+h)\mu_1. \tag{9}$$

From (7) and (8) we have

$$\mu_0^{0,S} = \frac{2n_0+\lambda}{2(n_0+\phi)}\mu_0^1; \tag{10}$$

$$\mu_0^{0,F} = \frac{2n_0+\lambda}{2n_0}\mu_0^1; \tag{11}$$

and from (9) we have that

$$\mu_1 = \frac{3(2n_0+\lambda)}{n_{-1}+h}\mu_0^1. \tag{12}$$

From (10), (11), (12) and the fact that in the steady state $\mu_1 + \mu_0^{0,F} + \mu_0^{0,S} + \mu_0^1 = 1$, we have

$$\mu_1 = \frac{6n_0(n_0+\phi)}{n_0(2n_0+\lambda)[6(n_0+\phi)+(n_{-1}+h)] + (n_0+\phi)(n_{-1}+h)(4n_0+\lambda)}; \tag{13}$$

$$\mu_0^{0,F} = \frac{(n_{-1}+h)(n_0+\phi)(2n_0+\lambda)}{n_0(2n_0+\lambda)[6(n_0+\phi)+(n_{-1}+h)] + (n_0+\phi)(n_{-1}+h)(4n_0+\lambda)}; \tag{14}$$

$$\mu_0^{0,S} = \frac{n_0(n_{-1}+h)(2n_0+\lambda)}{n_0(2n_0+\lambda)[6(n_0+\phi)+(n_{-1}+h)] + (n_0+\phi)(n_{-1}+h)(4n_0+\lambda)}; \tag{15}$$

$$\mu_0^1 = \frac{2n_0(n_0+\phi)(n_{-1}+h)}{n_0(2n_0+\lambda)[6(n_0+\phi)+(n_{-1}+h)] + (n_0+\phi)(n_{-1}+h)(4n_0+\lambda)}. \tag{16}$$

Since the aggregate flow of innovations (AI) is given by the sum of the outflows, from (13), (14), (15) and (16) we have

$$AI = \frac{2(n_0+\phi)(n_{-1}+h)[3n_0 + (2n_0+\lambda)(5n_0+2)]}{n_0(2n_0+\lambda)[6(n_0+\phi)+(n_{-1}+h)] + (n_0+\phi)(n_{-1}+h)(4n_0+\lambda)}.$$

Replacing the research intensities n_{-1} and n_0, we have the aggregate flow of innovations as a function of competition, memory, R&D spillovers and profit parameters. Since the research intensities do not have an analytical solution, next subsection shows the relationship between the aggregate flow of innovation and competition, using a numerical solution.

2.3 Innovation and competition relationship

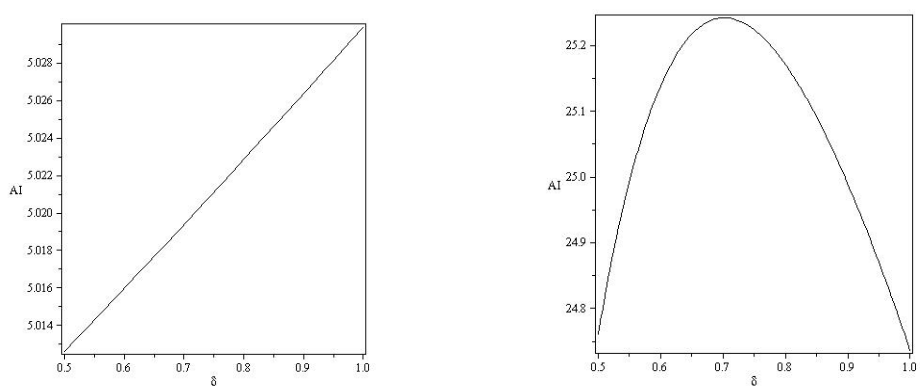
In order to solve this model and compare the results with the ABBGH model we have to follow the condition identified in ABBGH which satisfies the inverted-U relationship. First, I assume that the leader's profit π_1, the interest rate r and the time interval Δt are given. It is denoted $\lambda = \beta\pi_1$, $\phi = \alpha\pi_1$, and $h = \gamma\pi_1$.

Now, ABBGH define

$$\begin{aligned}
\widetilde{x} &\equiv \sqrt{\frac{2+\gamma^2\pi_1}{3}} + \gamma\sqrt{\pi_1}, \\
\underline{x} &\equiv \sqrt{1+\gamma^2\pi_1}, \\
\overline{x} &\equiv \sqrt{2+\gamma^2\pi_1}.
\end{aligned}$$

The inverted-U pattern holds whenever $\underline{x} < \widetilde{x} < \overline{x}$. Therefore, in order to construct the benchmark model I assume that $\alpha = 0.028$, $\beta = 0.04$, $\gamma = 0.018$, $\pi_1 = 500$, $r = 0.1$ and $\Delta t = 0.001$, parameters which satisfy the inverted-U condition[2].

Figure 2 shows the relationship between the aggregate flow of innovations and the competition parameter for both the memory and the ABBGH model. Subfigure 2a exhibits a clear positive relationship between innovation and competition.



(a) Memory Model

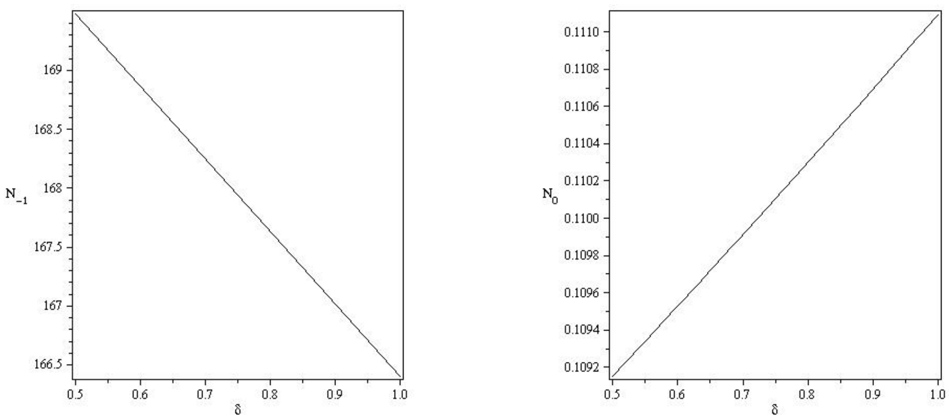
(b) ABBGH Model

Figure 2: Innovation and Competition

As we can see in figure 3, the Memory model also shows the Schumpeterian and escape-competition effects. As displayed in subfigure 3a, more innovation decreases the

[2]Figures 7, 8, 9 and 10 in Appendix A show the benchmark model's aggregate flows of innovation compare with the aggregate flows of innovation assuming other parameters. The modified assumptions appear in the title of each subfigure.

level of research intensity of the follower firm. However, even at the maximum level of competition the research intensity is still much higher than the research intensity of the neck-and-neck firm, as we can see in subfigure 3b.



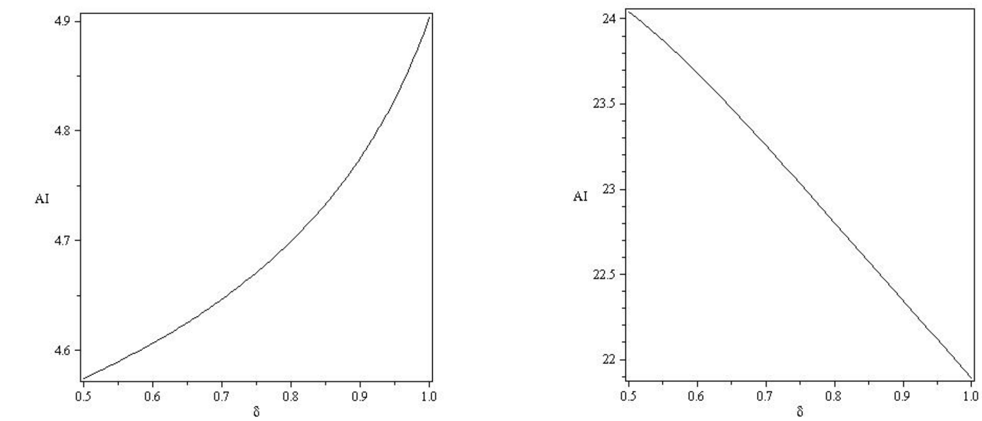
(a) Follower Research Intensity (b) Neck-and-Neck Research Intensity

Figure 3: Research Intensity

This is because whenever the follower innovates it will be in a better position than its rival when they will be neck-and-neck, since the former follower obtained the memory value λ after innovating while its rival's probability of innovating depends just on the research intensity. Since at any level of competition the research intensity of the follower is much higher than the neck-and-neck research intensity, the firms will be most of the time neck-and-neck, where the escape-competition effect dominates over the Schumpeterian effect.

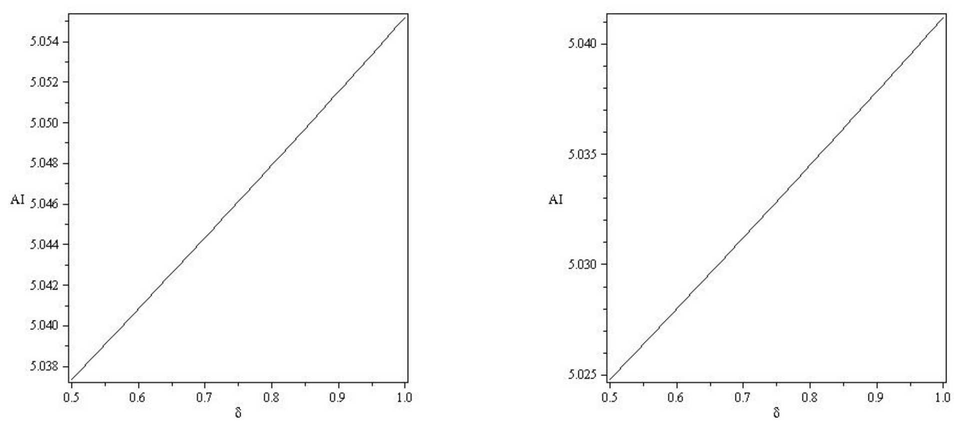
From ABBGH, the Schumpeterian effect will dominate over the whole interval whenever $\underline{x} \geq \widetilde{x}$. Therefore, with the same assumptions of the benchmark model but now with $\gamma = 0.002$, we can see the outcome in figure 4. Although in ABBGH model the Schumpeterian effect dominates in the whole interval, in the Memory model the relationship between innovation and competition is still positive.

In figure 5 we can see the effects of increasing the level of memory. Subfigures 5a and 5b show the effect of imposing the same assumptions of the benchmark model, but with $\alpha = 0.08$ and $\beta = 0.12$ respectively. We can see that the level of research intensity rises very little and the slope does not change.



(a) Memory Model (b) ABBGH Model

Figure 4: Schumpeterian Domination



(a) $\alpha = 0.08$ (b) $\beta = 0.12$

Figure 5: Increasing Memory

3 Empirical Relationship between Innovation and Competition

In the previous section it was shown that the theoretical inverted-U relationship between competition and innovation does not hold after assuming that innovation follows a memory process.

ABBGH empirical model shows an inverted-U relationship between innovation and competition. This pattern suggests that at low levels of competition, an increase in competition boosts innovation intensity until a threshold where it begins to decrease with more competition. ABBGH state that conditional citation weighted patents p_{jt} follow a Poisson process

$$p_{jt} = \exp\left\{\beta_0 + \beta_1 c_{jt} + \beta_2 c_{jt}^2 + \phi\hat{v}_{jt} + \sum_{j=22}^{49}\alpha_j D_j + \sum_{t=1973}^{1994}\gamma_t D_t + u_{jt}\right\}, \qquad (17)$$

where c_{jt} is 1 minus the Lerner Index (Competition Index) in industry j at time t, $\hat{v}$ denotes the vector of residuals from the regression of Competition Index on instruments, and the sums represent industry and time effects respectively[3].

3.1 Defining memory and memoryless innovation processes

It is turned now to matching this empirical finding to the theoretical model developed in the previous section. That theoretical model predicts a positive relationship between competition and innovation for industries where memory matters for innovation. Hence, in order to test this prediction, it is necessary to identify industries where memory plays a role in the innovation process.

As the ABBGH dataset does not have information about R&D investment, the memory path will be given by the relationship between contemporaneous and past conditional citation weighted patents.

Since Hausman et al. (1984) find that the negative binomial specification allows for over-dispersion, a problem which arises when using a Poisson in a memory process, it is employed an autoregressive model with drift and year trend, estimating the negative binomial regression

$$p_t^{(j)} = \exp\left\{\varsigma_0^{(j)} + \lambda^{(j)} t^{(j)} + \sum_{s=1}^{j}\varsigma_s^{(j)} p_{t-s}^{(j)} + \varepsilon_t^{(j)}\right\},$$

where $\varsigma_0^{(j)}$ is the drift for industry j, $t^{(j)}$ is the time trend of industry j and $p_{t-s}^{(j)}$ is the conditional citation weighted patents at the s lag. Following Hayashi (2000) to determine the memory in each industry, the sequential rule is used to test

$$\begin{aligned} H_0 &: \varsigma_j^{(i)} = 0, \\ H_1 &: otherwise. \end{aligned}$$

The test begins with a five-lags model. After testing significance of the last lag, it is dropped if not significant and the test is repeated recursively until $j = 1$. The outcome

[3] The sample is an unbalanced panel of 17 two-digit SIC code industries from 1973 to 1994. Industries and instruments included can be seen in tables 2 and 4 in Appendix B.

can be seen in table 3 in Appendix B. We can notice that the following industries show memory in the process of innovation: Extraction of other minerals (23), Chemicals (25), Office and computing machinery (33), Motor vehicles (35), and Food manufacture (41).

Even though Rubber and plastic products (48) industry does not show any significant coefficient for its lags, and Other manufacturing (49) industry exhibits memory, they are not considered in the subsamples, since more than 3/4 of the citation weighted patents in industry 48 are equal to zero and industry 49 has just 12 observations with half of them being equal to zero.

3.2 Innovation-competition relationship and its stability through time

Before 1982, appeals of patent cases were heard by the regional courts in the United States. However, after that year, all patent appeals have been analyzed by the Court of Appeals for the Federal Circuit. Jaffe and Lerner (2004) state that after the establishment of the CAFC there had been a significant increase in the number of patent applications as well as in the fraction of patents granted. They also state that the CAFC had produced a decrease in the level of quality of patents granted. Moser (2005) finds that the level of patent protection influences the direction of innovation activity. This suggests that the Reform of 1982 might have changed the incentives to patent, inducing industries which are more dependent of patent protection to be relatively more active than in the past.

Now, we proceed to analyze the structural break[4] which could be generated by the establishment of the CAFC. Performing a Chow test for both memory and memoryless samples defined as

$$p_{jt} = \exp\left\{ \beta_0 + \beta_1 c_{jt} + \beta_2 c_{jt}^2 + \phi \hat{v}_{jt} + \delta_1 D_\tau c_{jt} + \delta_2 D_\tau c_{jt}^2 + \sum_{j=22}^{49} \alpha_j D_j + \sum_{t=1973}^{1994} \gamma_t D_t + u_{jt} \right\},$$

where

$$D_\tau = \begin{cases} 1 & \forall t \geq \psi \\ 0 & \forall t < \psi \end{cases}$$

and testing

$$\begin{aligned} H_0 &: \delta_1 = \delta_2 = 0, \\ H_1 &: \textit{otherwise}; \end{aligned}$$

the null hypothesis of time stability is rejected at the 5% significance level for the memory sample, since the χ^2-statistic is 8.67. However, the null hypothesis rejection fails for the memoryless group, since the χ^2-statistic is 1.60.

[4] For further details about the structural break test, please contact the author.

Now, take into account this break for the memory sample and regressing equation (17), I proceed to test

$$\begin{aligned} H_0 &: \beta_1 = \beta_2 = 0, \\ H_1 &: \textit{otherwise}. \end{aligned}$$

which in the case of the memory sample leads to the null hypothesis rejection at 5% significance level for the period 1973-1982, but failing to reject it for the period 1983-1994, since the χ^2-statistics are 18.28 and 1.63, respectively. Besides, the null hypothesis rejection fails for the memoryless sample, since the χ^2-statistic is 4.46. Therefore, we can see the innovation-competition relationship is statistically significant only for the memory sample during the period 1973-1982.

The outcome of the regression (17) for both memory and memoryless group of industries can be seen in table 1.

Table 1: Memory and Memoryless

Citation Weighted Patents	Competition	Competition Squared	Constant	Industry Effects	Year Effects	Pseudo R^2	Observations
Memory 1973-1982	248.77 (159.44)	-122.78 (86.36)	-125.65 (73.36)	Yes	Yes	0.28	49
Memory 1983-1994	73.46 (114.89)	-41.42 (61.11)	-31.01 (54.05)	Yes	Yes	0.21	55
Memoryless 1973-1994	258.39 (160.41)	-140.15 (84.69)	-120.46 (76.04)	Yes	Yes	0.66	216

Standard errors in parentheses.

Figure 6 shows that for the memory sample the relationship between innovation and competition is positive for the period 1973-1982. Therefore, we can see the innovation-competition relationship during the period 1973-1982 follows a similar pattern than the theoretical model.

As stated by Jaffe and Lerner (2004), after the establishment of the CAFC there is a significant higher proportion of adjudicated patents to be confirmed as valid by judges. Consequently, we can expect that some patents of the period 1983-1994, which should have been considered *prior art* under the pre-CAFC context, would not be the outcome of an intensive research process and therefore they could not be a proper measure of innovation. This could explain the inconsistency of the innovation-competition relationship after splitting the sample between periods 1973-1982 and 1983-1994.

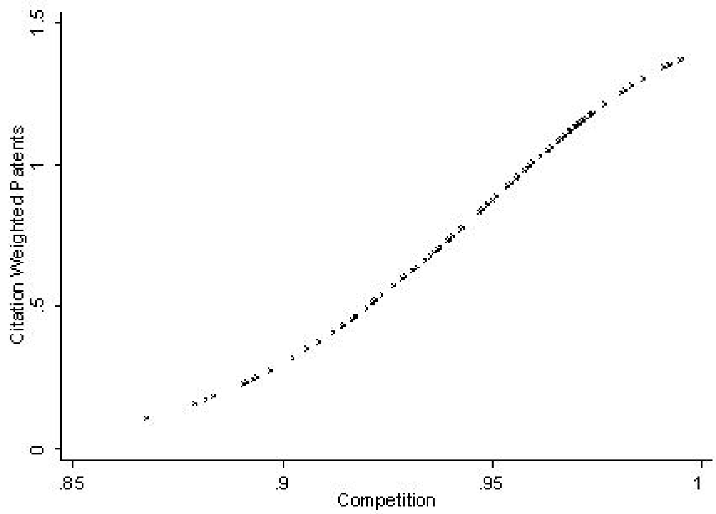


Figure 6: Memory 1973-1982

4 Conclusions

This paper analyzes the relationship between innovation and competition when the research activity follows a memory process. We have seen that whenever memory is assumed, there is a positive relationship between innovation and competition. A follower firm has very high incentive to innovate, even in a highly competitive environment, since the memory obtained after innovating increases its probability to innovate again and become a leader. Therefore, industries will be most of the time neck-and-neck where the escape-competition effect dominates.

We have also seen that empirical evidence supports this positive relationship during the period 1973-1982. However, after the establishment of the CAFC there is no significant relationship between innovation and competition. One possible explanation could be that the citation weighted patents variable would not be an accurate variable to measure innovation after the CAFC reform. However, further research should be done in order to prove it.

Both this paper's and ABBGH models assume that the leader cannot stay more than one step ahead of its rival. Aghion et al. (2001) relax this assumption, assuming that there is no bounding for the distance between the leader and the follower. However, they have found that there is no closed-form solution for the research equilibrium and the steady state industry structure. One extension for this model is to assume that the leader can be more than one step ahead from the follower, but binding the distance to a certain level.

This article assumes that the knowledge obtained through innovating is independent from the current period's research, thus this knowledge does not affect the marginal decision to invest in research. If this assumption is modified, the research intensities of the neck-and-neck who was leader, the neck-and-neck who was follower, neck-and-neck who failed to innovate and neck-and-neck who succeeded to innovate in the previous period, will be different. It would be interesting to test whether this modification changes the outcome of the model.

Another theoretical extension is to relax the assumption of short-memory. Instead of assuming that the research intensity depends just on the previous period's research, it can be the result of a cumulative process, more similar to the Doraszelski (2003) model.

There are also interesting extensions for the empirical model. We have seen, after considering the structural break, the inverted-U relationship does not hold. Although the 1973-1982 relationship is the same as the one predicted by the memory model, for the period 1983-1994 there is no significant relationship between innovation and competition. Therefore, it would be worthy to improve the sample in order to test whether this inconsistency remains.

Cohen et al. (2000) find a relatively low importance of patents to protect inventions. Moreover, they find that in some industries patents are used to block the development of rivals' products. Boone (2008) introduces an alternative procedure to measure competition, stating that it is more robust than the one used in ABBGH model. Consequently, further work can analyze innovation and competition relationship using different approaches to measuring these variables.

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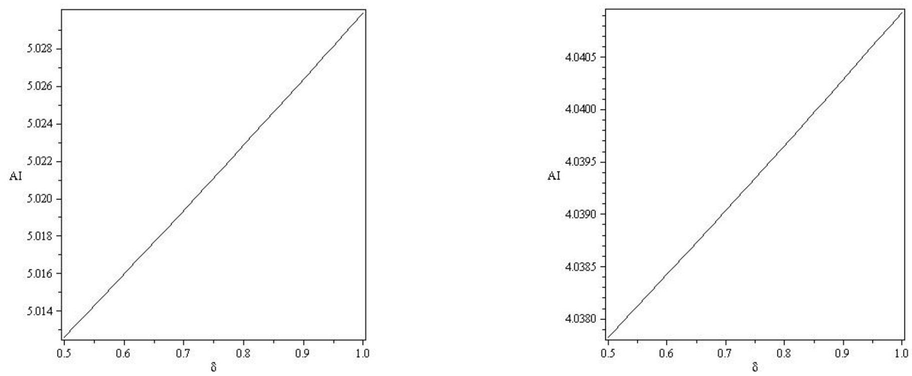
A Additional Figures

Figure 7 shows the case for very little memory, when both α and β are equal to 0.1%. We can see that the difference with the benchmark model is very small. When memory decreases the research intensity falls down while the slope is slightly flatter.

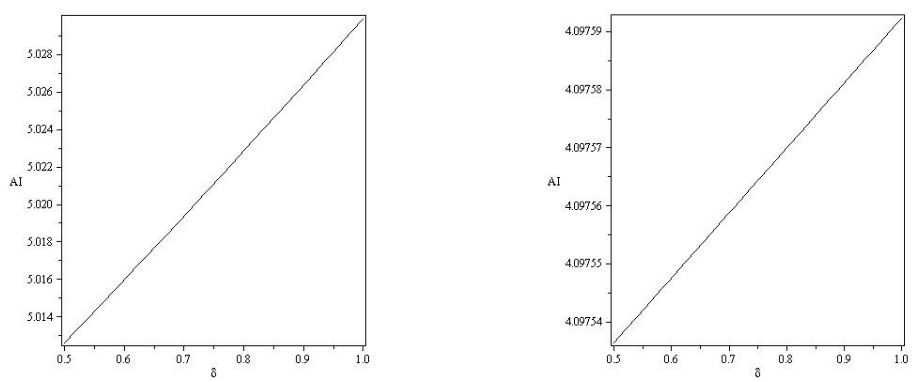


(a) Benchmark Model

(b) $\alpha = \beta = 0.001$

Figure 7: Memory Parameters

Figure 8 shows the case when decreasing the interest rate from 10% to 1%. As well as in the former figure the difference with the benchmark model is very small.

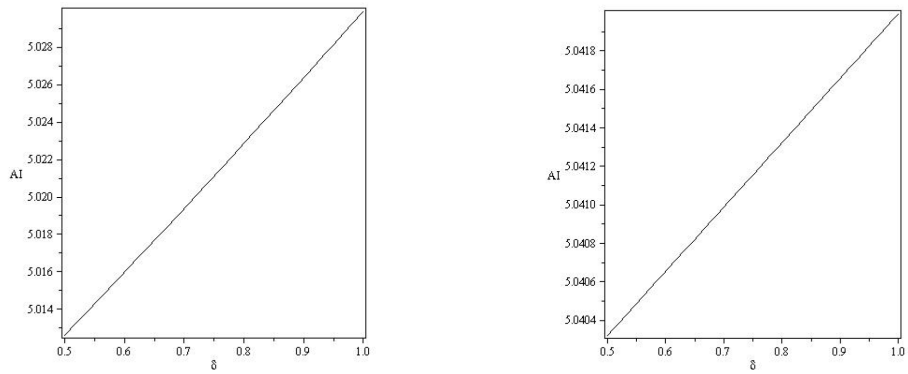


(a) Benchmark Model

(b) $r = 0.01$

Figure 8: Discount Rate

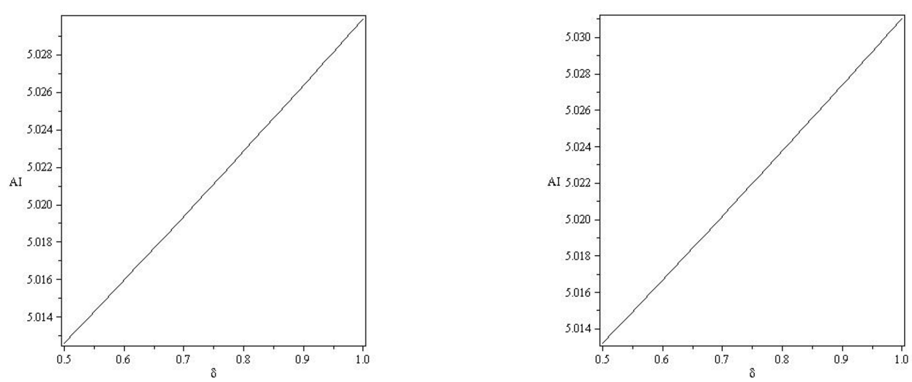
Figure 9 shows the case of a high copy rate. We can see now that when γ is equal to 6% the effect over the research intensity level is almost unperceptible and the relationship between innovation and competition is also positive.



(a) Benchmark Model (b) $\gamma = 0.06$

Figure 9: R&D Spillovers

Figure 10 shows the case when the time interval Δt decreases from 0.1% to 0.01%. There is almost no difference with the benchmark model.



(a) Benchmark Model (b) $\Delta t = 0.0001$

Figure 10: Time Interval

B Additional Tables

Table 2: Industries

SIC Code	Industry
22	Metal Manufacturing
23	Extraction of Other Minerals
24	Non-Metallic Mineral Products
25	Chemicals
31	Manufacture of Metal Goods
32	Mechanical Engineering
33	Office & Computing Machinery
34	Electrical & Electronic Engineering
35	Motor Vehicles
36	Manufacture of Other
37	Instrument Engineering
41	Food Manufacture
42	Sugar Beverages & Tobacco
43	Textiles
47	Paper, Paper Products & Printing
48	Rubber & Plastic Products
49	Other Manufacturing

Table 3: Defining Memory and Memoryless Industries

Industry	5th Lag	4th Lag	3rd Lag	2nd Lag	1st Lag
22	-0.17	-0.07	-0.08	-0.00	0.02
	(1.18)	(0.33)	(0.45)	(0.00)	(0.05)
23	-0.20	-0.16	0.16	-0.19	0.22
	(2.37)	(5.97)*	(9.92)**	(8.41)**	(6.94)**
24	-0.05	-0.03	0.00	0.06	0.02
	(0.31)	(0.12)	(0.00)	(0.57)	(0.04)
25	-0.02	-0.01	-0.00	0.01	0.02
	(0.77)	(0.43)	(0.04)	(0.32)	(4.44)*
31	-2.06	-4.98	-1.06	2.21	4.18
	(0.04)	(0.22)	(0.01)	(0.07)	(0.35)
32	-0.47	-0.27	-0.05	0.03	0.28
	(0.49)	(0.21)	(0.01)	(0.00)	0.27
33	-0.00	-0.03	0.00	0.02	0.06
	(-.00)	(0.84)	(0.00)	(0.52)	(4.79)*
34	-0.01	-0.08	0.01	0.00	0.04
	(0.01)	(1.23)	(0.03)	(0.00)	(0.40)
35	0.01	-0.01	0.03	-0.01	0.03
	(0.06)	(0.13)	(2.31)	(0.13)	(15.86)**
36	-0.05	0.04	0.03	-0.00	0.02
	(1.76)	(0.98)	(0.55)	(0.01)	(0.20)
37	-0.09	-0.10	-0.12	-0.28	-0.14
	(0.01)	(0.11)	(0.15)	(0.87)	(0.34)
41	-0.21	0.04	0.04	0.02	0.11
	(4.50)*	(0.39)	(0.42)	(0.12)	(4.97)*
42	-0.01	-0.01	-0.02	0.02	0.00
	(0.45)	(0.23)	(1.36)	(1.74)	(0.02)
43	0.34	-0.73	-0.51	-1.14	0.40
	(0.02)	(0.11)	(0.05)	(0.24)	(0.04)
47	0.05	NC	0.34	NC	0.09
	(0.00)		(0.03)		(0.00)
48	ZA	NC	NC	NC	-0.50
					(0.74)
49	-0.01	NC	NC	0.10	0.23
	(0.58)			(3.89)*	(4.56)*

Standard errors in parentheses. ** significant at 1% level; * significant at 5% level.

NC non-concavity of likelihood function; ZA dependent variable is zero for all observations.

Table 4: Instruments

Number	Instrument	Type	Number	Instrument	Type
1	Industry R&D/ Y USA	Foreign-industry	19	Markup USA squared	Foreign-industry
2	Industry R&D/ Y USA squared	Foreign-industry	20	Output - variable costs/ Output USA	Foreign-industry
3	Industry TFP USA	Foreign-industry	21	Output - variable costs/ Output USA squared	Foreign-industry
4	Industry TFP USA squared	Foreign-industry	22	Markup France	Foreign-industry
5	Industry R&D/ Y France	Foreign-industry	23	Markup France squared	Foreign-industry
6	Industry R&D/ Y France squared	Foreign-industry	24	Output - variable costs/ Output France	Foreign-industry
7	Industry TFP France	Foreign-industry	25	Output - variable costs/ Output France squared	Foreign-industry
8	Industry TFP France squared	Foreign-industry	26	SMP high impact	Policy
9	TFP France dummy for SIC2 49	Foreign-industry	27	SMP medium impact	Policy
10	Industry imports/ Y USA	Foreign-industry	28	Car industry	Policy
11	Industry imports/ Y USA squared	Foreign-industry	29	Periodicals industry	Policy
12	Industry imports/ Y France	Foreign-industry	30	Brewing industry	Policy
13	Industry imports/ Y France squared	Foreign-industry	31	Telecoms industry	Policy
14	Industry exports/ Y USA	Foreign-industry	32	Pharmaceuticals industry	Policy
15	Industry exports/ Y USA squared	Foreign-industry	33	Textiles industry	Policy
16	Industry exports/ Y France	Foreign-industry	34	Razor industry	Policy
17	Industry exports/ Y France squared	Foreign-industry	35	Steel industry	Policy
18	Markup USA	Foreign-industry	36	Ordnance industry	Policy